Transient Electromagnetic Analysis and Model Complexity Reduction Using the Partial Element Equivalent Circuit Formulation

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***Abstract*—Nonlinear electromagnetic field simulation is becoming essential for mixed signal (RF/microwave and digital) circuit design applications. So far, direct analysis of such nonlinear electromagnetic circuits has been demonstrated successfully only within the context of the the FDTD and the TLM method. In this paper, it is shown that time-domain integral equation techniques can be used also for nonlinear electromagnetic circuit analysis. In particular, the application of the so-called Partial Element Equivalent Circuit formulation of the time-domain electric field integral equation leads to a very convenient SPICE-compatible approximation of the electromagnetic problem. Furthermore, it has the important attribute of lending itself to a very systematic and physical model complexity reduction on the basis of the electrical size of the various portions of the system under study. Numerical experiments are used to demonstrate these special atrributes of this formulation.**

I. Introduction

One of the most useful applications of transient electromagnetic field solvers in the analysis of linear microwave/millimeter wave circuits has been in the efficient, broadband scattering parameter characterization of passive multiport circuits of significant geometric complexity. Another important application of such transient simulators that is receiving significant attention recently is the analysis of nonlinear electromagnetic systems. Such modeling capability is important for nonlinear RF/microwave circuit analysis, mixed (analog/digital) distributed circuit analysis, and for integrated circuit electromagnetic circuit analysis. Both the FDTD method and the TLM method have been shown to be suitable for such nonlinear electromagnetic simulations [1]-[4].

Time-domain integral equation-based formulations are also suitable for nonlinear electromagnetic circuit analysis. The reasons for the numerical instability exhibited by such formulations are now well understood, and several approaches have been proposed for its prevention (e.g. [5]-[8]). Of particular interest to RF/microwave and high-frequency, high-speed, mixed-signal circuits is the Partial Element Equivalent Circuit (PEEC) formulation of the time-domain electric field integral equation [9]. This formulation results in a circuit model for the electromagnetic problem under study, which includes all retardation effects and is compatible with SPICE-like circuit simulators. Consequently full-wave electromagnetic modeling within a SPICE-like, nonlinear, circuit simulation environment can be effected in a straightforward fashion.

In this paper, the development of circuit simulator-compatible PEEC approximations to distributed electromagnetic systems is presented. The emphasis of the presentation is placed on the scheme used for stabilizing the transient numerical integration and on the suitability of the PEEC formulation for systematic model complexity reduction. This latter attribute of the PEEC formulation is extremely useful when the size or complexity of the structure under study is such that the number of degrees of freedom used in the numerical approximation is much larger than the one required for acceptable engineering accuracy of the simulation results. Numerical examples will be used to illustrate the importance of this attribute of PEEC and its impact on the reduction of the simulation complexity.

II. Development of PEEC Models

As discussed in detail in [9], the development of the PEEC approximation of an electromagnetic boundary value problem is based on the proper electromagnetic interpretation of the various terms in the equation

$$\vec{E}(\vec{r},t) = -\frac{\partial \vec{A}(\vec{r},t)}{\partial t} - \nabla\Phi(\vec{r},t) \tag{1}$$

with the potential terms written in integral form in terms of the spatial and temporal current and charge distributions in the conductors. The discretization of the conductors is effected using pulse basis functions for the current distributions and the charges. When dielectrics are present, polarization currents are introduced [10]. This way, for the case of conductors of

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finite extent, free-space Green's functions are used in the integral representation of the potentials. Standard Galerkin's testing is used to obtain the matrix approximation of the time-domain electric field integral equation and the development of the PEEC representation of the structure.

Figure 1 depicts the PEEC circuit for two adjacent conducting cells of zero thickness. The partial inductances $L_{p_{ij}}$ and coefficients of potential p_{ij} are computed according to [11] and [12], respectively. Retardation is taken into account in the coupling between the inductors and between the capacitors, thus maintaining the full-wave nature of the problem. From Fig. 1 it is clear that the introduction of lumped circuit blocks in the PEEC model is straightforward. In particular, the use of the condensed MNA matrix formulation for the state representation of the PEEC model makes it compatible with SPICE-like circuit simulators. The general form of the matrix for the PEEC model in the Laplace domain is

$$\begin{bmatrix} \mathbf{Y} & (\mathbf{P}-\mathbf{A}) \\ \mathbf{A}^T & \mathbf{Z} \end{bmatrix} \begin{bmatrix} \mathbf{V} \\ \mathbf{I} \end{bmatrix} = \begin{bmatrix} \mathbf{I_s} \\ \mathbf{V_s} \end{bmatrix} \tag{2}$$

where $\mathbf{V}$ are the nodal voltages, which for the electromagnetic case correspond to the potentials to infinity, $\mathbf{I}$ are the inductance branch currents, and $\mathbf{I_s}$ and $\mathbf{V_s}$ are source terms including both point excitation and distributed excitation (e.g. incident electromagnetic fields). The admittance matrix $\mathbf{Y}$ includes the capacitances while the impedance matrix $\mathbf{Z}$ includes the partial inductances. The matrix $\mathbf{A}$ is the incidence matrix of the resulting circuit.The presence of retardation implies that terms of the form $exp(-s\tau)$ are associated with the off-diagonal terms in Y and Z, where τ denotes time delay.

III. Model Complexity Reduction and Examples

As mentioned in the introduction, once the PEEC model is developed, a systematic procedure can be used to reduce its complexity taking into account the characteristic times over which the system response is sought or, equivalently the electrical size of the discretized structure. For example, if the characteristic time of the excitation (i.e. the rise time of a pulsed excitation or the period of a sinusoidal excitation) is such that the useful wavelengths are much larger than the spatial extent of the system, all retardation effects can be neglected. As another example, if the electromagnetic system under investigation exhibits transmission line behavior, retardation effects in the transverse direction can be neglected and maintained only along the axis of the interconnect. Or, even simpler, if the frequency range of interest is appropriate, the typical SPICE model for the transmission line based on the method of characteristics can be used. More importantly, such model complexity reduction can be effected in a piecewise fashion for different parts of a complex structure.

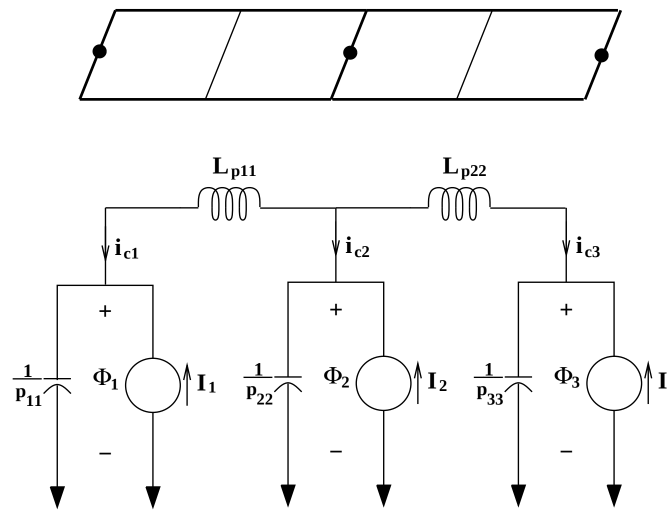

Fig. 1. Simple PEEC model for two adjacent conducting cells.

As an example, let us consider the case of a coplanar waveguide structure of length 1.8 cm. The center and ground strips are assumed perfectly conducting, infinitesimally thin and of width 1 mm. The separation of the center strip from the ground conductors is 0.5 mm. The waveguide is driven by a current source at one end and terminated at a short at the other end. Figure 2 compares the imaginary part of the input voltage versus frequency as calculated using the method of characteristics and the full-wave PEEC model. The agreement is excellent at the lower frequencies, as expected. However, at higher frequencies the end effects associated with the finite extent of the waveguide start becoming important, as indicated by the downward shift of the PEEC response.

The second example deals with a right-angle bent discontinuity in a coplanar waveguide (Fig. 3). Often times, a microwave circuit model for such a structure is constructed by introducing either a lumped equivalent circuit or an s-parameter based representation of the discontinuity between two sections of uniform transmission line on either end. Such models are suitable only if the waveguiding structure supports a single propagating mode over the bandwidth of interest and if the discontinuity does not introduce a significant imbalance in the return path. Clearly, this is not the case of the structure in Fig. 3. The two ground electrodes are of unequal lengths. Furthermore, slot line

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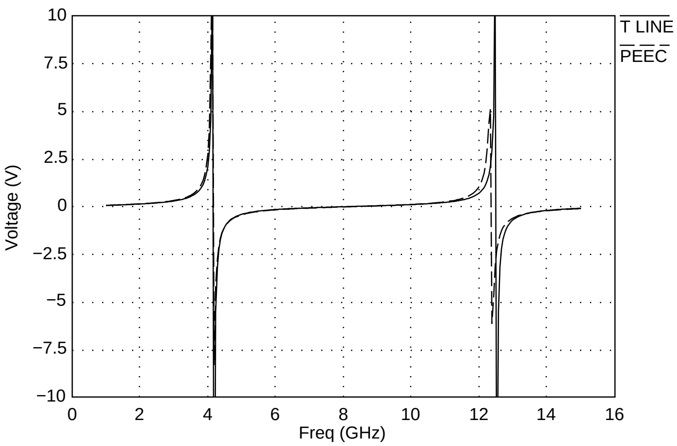

Fig. 2. Comparison of transmission line and full-wave PEEC responses for a uniform section of a short-circuited coplanar waveguide.

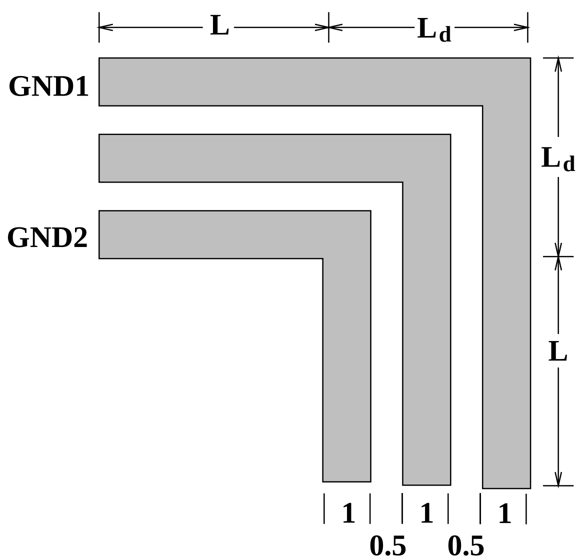

Fig. 3. A coplanar waveguide with a right-angle bend.

modes can be supported by this structure. Therefore, one expects that only a full-wave model can be used to capture the correct electromagnetic behavior of this discontinuity.

The PEEC formulation facilitates the straightforwrad investigation of the above conjecture. The structure of Fig. 3 with $L = 2$ cm and $L_d = 0.4$ cm was driven by a voltage source at one end and was short-circuited at the other end. Figure 4 depicts the imaginary part of the two ground conductor currents at the input. The ground current imbalance is clearly demonstrated. This imbalance makes the approximation of this planar structure in terms of a transmission line-based model with some type of equivalent representation of the discontinuity impossible. To demonstrate

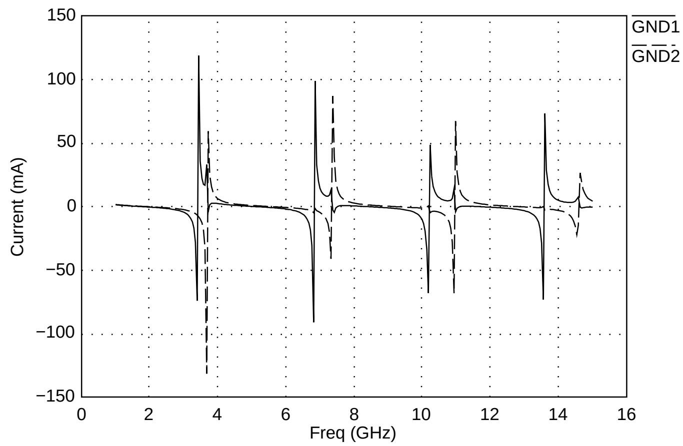

Fig. 4. Imaginary part of the ground currents at the input for the coplanar waveguide structure of Fig. 3.

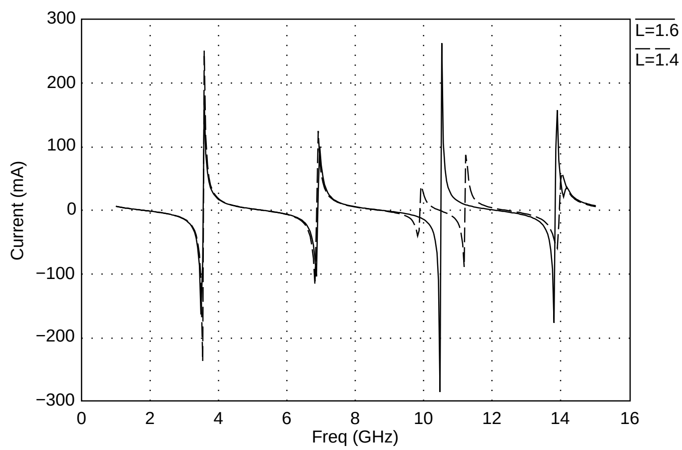

Fig. 5. Imaginary part of the current at the input of the transmission line for the approximate transmission line-based representation of the structure of Fig. 3.

this, we calculated the imaginary part of the input current for a PEEC model that used a full-wave PEEC for the discontinuity region represented by the lengths L_d, and transmission line models implemented using the method of characteristics for the lengths L on either side. The results are depicted in Fig. 5 for two cases: a) $L = 1.6$ cm, $L_d = 0.8$; b) $L = 1.4$ cm, $L_d = 1.0$ cm. Clearly, the responses are very different from the one in Fig. 4, illustrating the inadequacy of these simpler, approximate models.

Finally, as an example from the application of the method for transient analyis, the coplanar waveguide structure of Fig. 2 was driven by a trapezoidal voltage pulse of rise and fall times of 0.1 ns, duration 0.3 ns and amplitude 1 V. Two resistors, 268 ohms each, are connected are connected between the center strip and each of the two ground conductors. With the characteristic impedance of the uniform coplanar line equal

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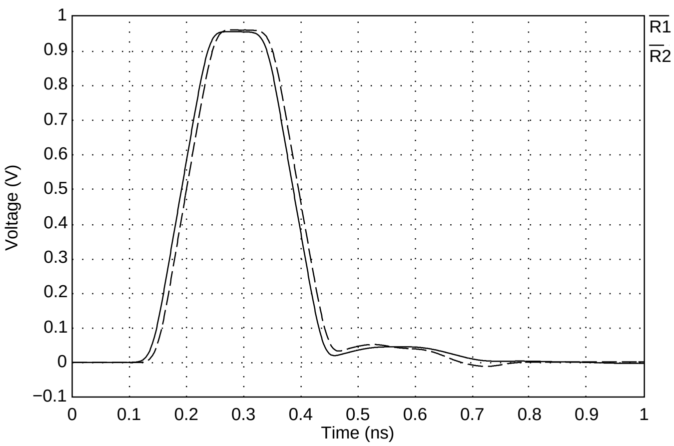

Fig. 6. Transient voltage responses across the two resistors used to terminate the structure of Fig. 3.

to 134 ohms, these resistors serve as an almost matched termination. Figure 5 depicts the voltages across these two resistors. Their slight difference is due to the aforementioned assymetry of the structure. The presence of the bend gives rise to significant distortion and broadening of the pulse.

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